

# REDUCE

User's and  
Contributed Packages Manual  
Version 3.8

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## Preface

This volume has been prepared by Codemist Ltd. from the  $\text{\LaTeX}$  documentation sources distributed with REDUCE 3.8. It incorporates the User's Manual, and documentation for all the User Contributed Packages as a second Part. A common index and table of contents has been prepared. We hope that this single volume will be more convenient for REDUCE users than having two unrelated documents. Particularly in Part 2 the text of the authors has been extensively edited and modified and so the responsibility for any errors rests with us.

Parts I and III were written by Anthony C. Hearn. Part II is based on texts by:

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# Contents



## Part I

# REDUCE User's Manual





# REDUCE

User's Manual

Version 3.8

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# Abstract

This document provides the user with a description of the algebraic programming system REDUCE. The capabilities of this system include:

1. expansion and ordering of polynomials and rational functions,
2. substitutions and pattern matching in a wide variety of forms,
3. automatic and user controlled simplification of expressions,
4. calculations with symbolic matrices,
5. arbitrary precision integer and real arithmetic,
6. facilities for defining new functions and extending program syntax,
7. analytic differentiation and integration,
8. factorization of polynomials,
9. facilities for the solution of a variety of algebraic equations,
10. facilities for the output of expressions in a variety of formats,
11. facilities for generating numerical programs from symbolic input,
12. Dirac matrix calculations of interest to high energy physicists.



# Acknowledgment

The production of this version of the manual has been the result of the contributions of a large number of individuals who have taken the time and effort to suggest improvements to previous versions, and to draft new sections. Particular thanks are due to Gerry Rayna, who provided a draft rewrite of most of the first half of the manual. Other people who have made significant contributions have included John Fitch, Martin Griss, Stan Kameny, Jed Marti, Herbert Melenk, Don Morrison, Arthur Norman, Eberhard Schröder, Larry Seward and Walter Tietze. Finally, Richard Hitt produced a  $\text{\TeX}$  version of the REDUCE 3.3 manual, which has been a useful guide for the production of the  $\text{\LaTeX}$  version of this manual.



# Chapter 1

## Introductory Information

REDUCE is a system for carrying out algebraic operations accurately, no matter how complicated the expressions become. It can manipulate polynomials in a variety of forms, both expanding and factoring them, and extract various parts of them as required. REDUCE can also do differentiation and integration, but we shall only show trivial examples of this in this introduction. Other topics not considered include the use of arrays, the definition of procedures and operators, the specific routines for high energy physics calculations, the use of files to eliminate repetitious typing and for saving results, and the editing of the input text.

Also not considered in any detail in this introduction are the many options that are available for varying computational procedures, output forms, number systems used, and so on.

REDUCE is designed to be an interactive system, so that the user can input an algebraic expression and see its value before moving on to the next calculation. For those systems that do not support interactive use, or for those calculations, especially long ones, for which a standard script can be defined, REDUCE can also be used in batch mode. In this case, a sequence of commands can be given to REDUCE and results obtained without any user interaction during the computation.

In this introduction, we shall limit ourselves to the interactive use of REDUCE, since this illustrates most completely the capabilities of the system. When REDUCE is called, it begins by printing a banner message like:

```
Reduce (Free CSL version), 25-Oct-14 ...
```

where the version number and the system release date will change from time to time. It then prompts the user for input by:

1:

You can now type a REDUCE statement, terminated by a semicolon to indicate the end of the expression, for example:

$(x+y+z)^2;$

This expression would normally be followed by another character (a Return on an ASCII keyboard) to “wake up” the system, which would then input the expression, evaluate it, and return the result:

$$X^2 + 2*XY + 2*XZ + Y^2 + 2*YZ + Z^2$$

Let us review this simple example to learn a little more about the way that REDUCE works. First, we note that REDUCE deals with variables, and constants like other computer languages, but that in evaluating the former, a variable can stand for itself. Expression evaluation normally follows the rules of high school algebra, so the only surprise in the above example might be that the expression was expanded. REDUCE normally expands expressions where possible, collecting like terms and ordering the variables in a specific manner. However, expansion, ordering of variables, format of output and so on is under control of the user, and various declarations are available to manipulate these.

Another characteristic of the above example is the use of lower case on input and upper case on output. In fact, input may be in either mode, but output is usually in lower case. To make the difference between input and output more distinct in this manual, all expressions intended for input will be shown in lower case and output in upper case. However, for stylistic reasons, we represent all single identifiers in the text in upper case.

Finally, the numerical prompt can be used to reference the result in a later computation.

As a further illustration of the system features, the user should try:

`for i:= 1:40 product i;`

The result in this case is the value of 40!,